Introduction to Quantitative Geology

Advection of the Earth’s surface:
Fluvial incision and rock uplift

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20.11.2017
Goals of this lecture

• Introduce the advection equation

• Discuss application of the advection equation to bedrock river erosion
What is advection?

- Advection involves a lateral translation of some quantity
- For example, the transfer of heat by physical movement of molecules or atoms within a material. A type of convection, mostly applied to heat transfer in solid materials.

http://homepage.usask.ca/~sab248/
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Diffusion equation

- Last week we were introduced to the **diffusion equation**

\[ q = -\rho \kappa \frac{\partial h}{\partial x} \]

- Flux (transport of mass or transfer of energy) proportional to a gradient

\[ \frac{\partial h}{\partial t} = -\frac{1}{\rho} \frac{\partial q}{\partial x} \]

- Conservation of mass: *Any change in flux results in a change in mass/energy*
Diffusion equation

\[ \frac{\partial h}{\partial t} = -\kappa \frac{\partial^2 h}{\partial x^2} \]

- Substitute the upper equation on the left into the lower to get the classic \textbf{diffusion equation}

- \( q = \text{sediment flux per unit length} \)
- \( \rho = \text{bulk sediment density} \)
- \( \kappa = \text{sediment diffusivity} \)
- \( h = \text{elevation} \)
- \( x = \text{distance from divide} \)
- \( t = \text{time} \)
Advection and diffusion equations

Diffusion
\[
\frac{\partial h}{\partial t} = -\kappa \frac{\partial^2 h}{\partial x^2}
\]

Advection
\[
\frac{\partial h}{\partial t} = c \frac{\partial h}{\partial x}
\]

- This week we meet the advection equation
Advection and diffusion equations

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Advection
\[ \frac{\partial h}{\partial t} = c \frac{\partial h}{\partial x} \]

- This week we meet the advection equation
- Two key differences:
  - Change in mass/energy with time proportional to gradient, rather than curvature (or change in gradient)
  - Advection coefficient \( c \) has units of \([L/T]\), rather than \([L^2/T]\)
Advection and diffusion equations

River channel profiles

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Fig. 1.7, Pelletier, 2008
Fig. 1.6 (a) Shaded-relief image of the Hanaupah Canyon drainage network and alluvial fan. Location of longitudinal profile shown as white curve. (b) Longitudinal profile of main Hanaupah Canyon channel, together with best-fit to Eq. (1.8).

Fig. 1.7 Schematic diagrams of the evolution of (a) bedrock and (b) alluvial channels through time, illustrating the advective behavior of bedrock channels and the diffusive behavior of alluvial channels.

The Kern River (Figures 1.8 and 1.9) provides a nice example of knickpoint propagation in action. Two distinct topographic surfaces have long been recognized in the landscape of the southern Sierra Nevada (Webb, 1946) (Figure 1.8). The Boreal surface is a high-elevation, low-relief plateau that dips to the west at $1^\circ$ (Figure 1.8b). The Chagoopa Plateau is an intermediate topographic ''bench'' that is restricted to the major river canyons and inset into the Boreal Plateau (Webb, 1946; Jones, 1987). Figure 1.8b maps the maximum extents of the Chagoopa and Boreal Plateaux based on elevation ranges of 1750--2250 m (Chagoopa) and 2250--3500 m a.s.l. (Boreal). Associated with each surface are prominent knickpoints along major rivers. Knickpoints along the North Fork Kern River, for example (Figure 1.9b), occur at elevations of 1600--2100 m and 2500--3300 m a.s.l. The stepped nature of the Sierra Nevada topography is generally considered to be the result of two pulses of Cenozoic and/or late Cretaceous uplift (Clark et al., 2005; Pelletier, 2007c). According to this model, two major knickpoints were created during uplift, each initiating a wave of incision that is still propagating headward towards the range crest.

Recent work has highlighted the importance of abrasion in controlling bedrock channel evolution. In the abrasion process it is sediment, not water, that acts as the primary erosional agent. In the stream-power model, the erosive power is assumed to be a power function of drainage area. Although sediment flux increases with drainage area, upstream relief also plays an important role in controlling sediment flux. As such, the stream-power model does not adequately represent the abrasion process. Sklar and Dietrich (2001, 2004) developed a saltation-abrasion model to quantify this process of bedrock channel erosion. Insights into their model can be gained by replacing drainage area with sediment flux in the stream-power model.
Advection and diffusion equations

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### Diffusion

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### Advection

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Advection and diffusion equations

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- **Diffusion**: Rate of erosion depends on change in hillslope gradient (curvature)
- **Advection**: Rate of erosion is directly proportional to hillslope gradient
- Also, no conservation of mass (deposition)

Fig. 1.7, Pelletier, 2008
Advection at a constant rate $c$

- Surface elevation changes in direct proportion to surface slope.
- Result is lateral propagation of the topography or river channel profile.
- Although this is interesting, it is not that common in nature.
Advection of the Earth’s surface: An example

- Bedrock river erosion
- Purely an advection problem with a spatially variable advection coefficient
Bedrock river erosion

- Not much bedrock being eroded here…
Bedrock river erosion

- Rapid bedrock incision has formed a steep gorge in this case

Kali Gandaki river gorge, central Nepal
http://en.wikipedia.org/
River erosion as an advection process

- With a constant advection coefficient $c$, we predict lateral migration of the river profile at a constant rate $(c)$
River erosion as an advection process

- With a constant advection coefficient \( c \), we predict **lateral migration of the river profile at a constant rate** (\( c \))
- Do you think this works in real (bedrock) rivers?
River erosion as an advection process

- With a constant advection coefficient $c$, we predict lateral migration of the river profile at a constant rate $(c)$
- Do you think this works in real (bedrock) rivers?
- What might affect the rate of lateral migration?
What affects the efficiency of river erosion?

- The amount of water flowing in the river (discharge) and sediment
- The slope of the river channel
- The strength of the underlying bedrock
What affects the efficiency of river erosion?

- The **amount of water flowing** in the river (discharge) and sediment
- The **slope** of the river channel
- The **strength of the underlying bedrock**

- **Are these constant?**
Stream-power model of river incision

Rather than being constant, the rate of lateral advection in river systems is spatially variable

\[ \frac{\partial h}{\partial t} = k_f \frac{Q}{w} \frac{\partial h}{\partial x} \]

where \( k_f \) is a material property of the bedrock (erodibility), \( w \) is the channel width, and \( Q \) is discharge
Stream-power model of river incision

• Rather than being constant, the rate of lateral advection in river systems is spatially variable

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\frac{\partial h}{\partial t} = c \frac{\partial h}{\partial x}
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where \( k_f \) is a material property of the bedrock (erodibility), \( w \) is the channel width, and \( Q \) is discharge

• This is known as the stream-power erosion model
Stream-power model of river incision

• If we assume precipitation is uniform in the drainage basin, discharge $Q$ will scale with drainage basin area, so we can modify our equation to read

$$\frac{\partial h}{\partial t} = \frac{k_f}{w} Q \frac{\partial h}{\partial x} \rightarrow \frac{\partial h}{\partial t} = KA^m S^n$$

where $K$ is an erosional efficiency factor (accounts for lithology, climate, channel geometry, sediment supply, etc. (!)), $A$ is upstream drainage area, $S$ is channel slope, and $m$ and $n$ are area and slope exponents.

• If we assume the drainage basin area increases with distance from the drainage divide $x$, we can replace the area with an estimate $A = x^{5/3}$
Test your might

\[ \frac{\partial h}{\partial t} = U - KA^m S^n \]

- Based on our **stream-power erosion** equation, what general form would a channel profile take?

- If we assume we have reached a steady state \((\partial h/\partial t = 0)\) and \(n = 1\), erosion must balance uplift \(U\) everywhere.

- If we further assume precipitation is constant, bedrock erodibility is constant and \(A = x^{5/3}\), how would the channel steepness vary as you move downstream from the divide?

- Think about how \(S\) would change as \(x\) increases.
Evolution of a channel profile

- A few stream-power erosion observations:
  - Stream power increases downstream as the discharge grows
  - Steeper slopes occur upstream where the discharge is low
  - Incision migrates upstream until a balance is attained between erosion and uplift

Fig. 3.23, Allen, 1997
Recap

- **What is the main difference between the advection and diffusion equations?**

- **What is special about the stream power erosion model compared to the general advection equation?**
Recap

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• What is special about the stream power erosion model compared to the general advection equation?
References
