Introduction to Quantitative Geology

Advection of the Earth's surface: Fluvial incision and rock uplift

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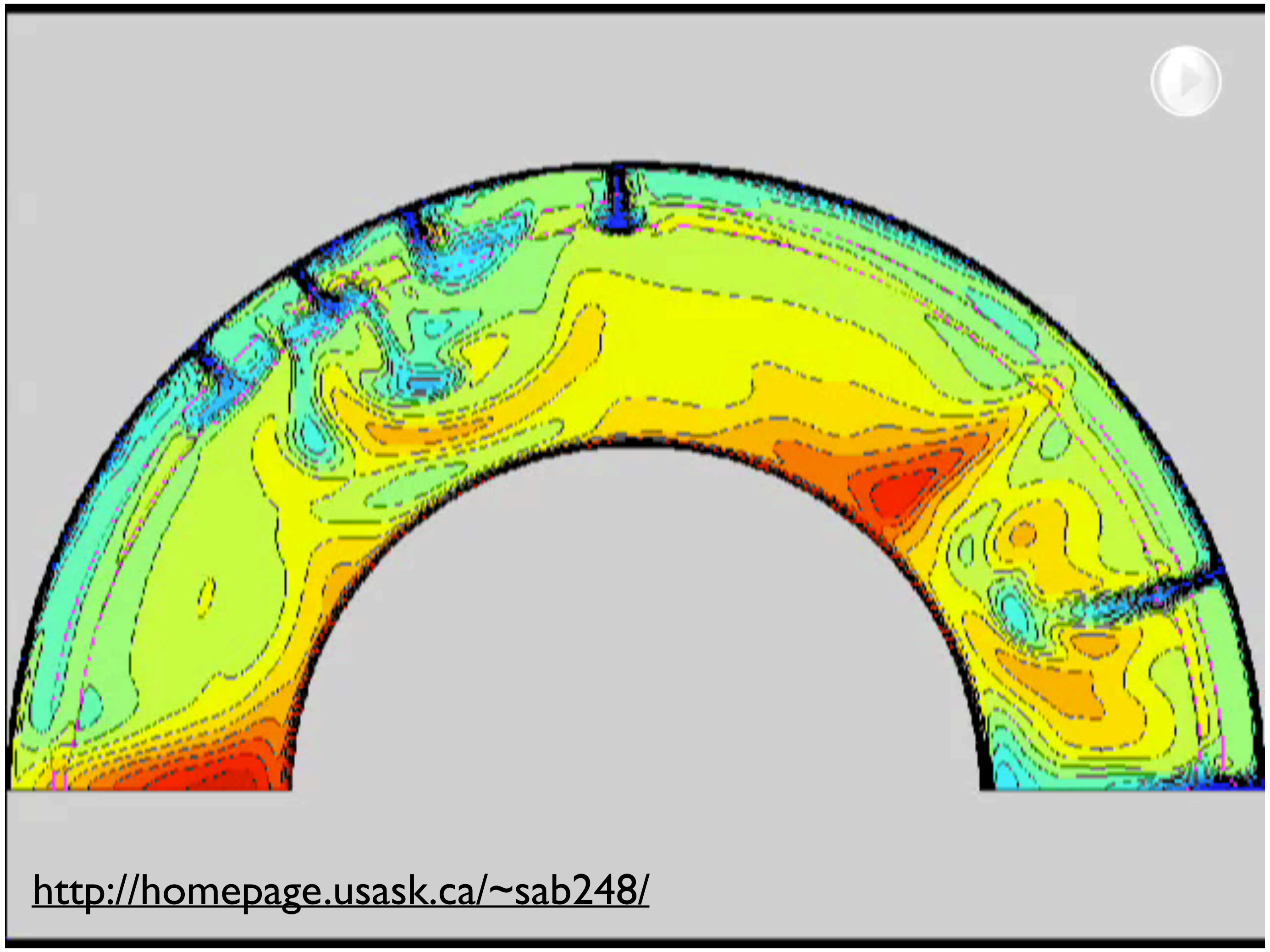
Goals of this lecture

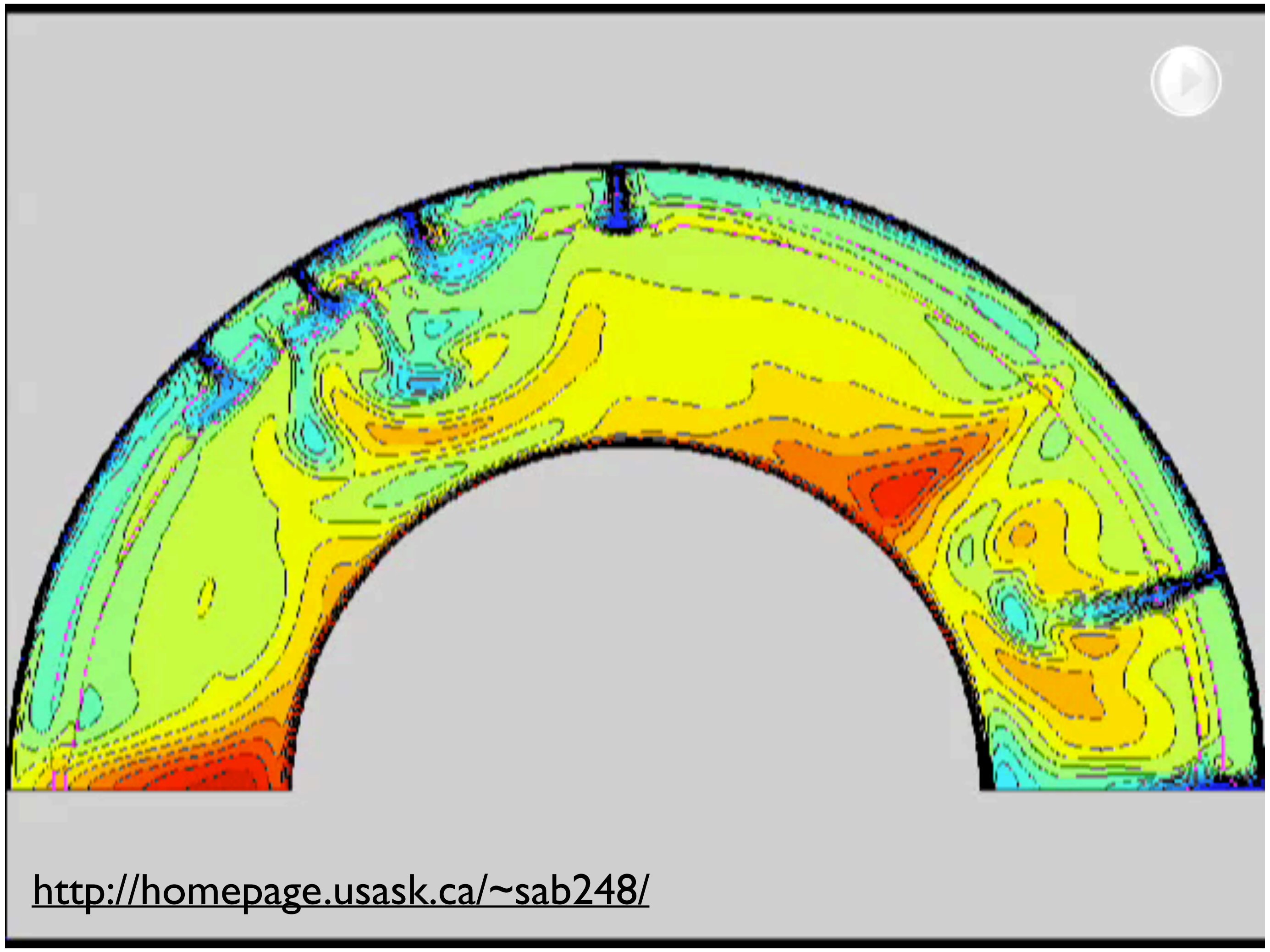
- Introduce the **advection equation**
- Discuss application of the advection equation to **bedrock river erosion**

What is advection?



- Advection involves a lateral translation of some quantity
  - For example, the transfer of heat by physical movement of molecules or atoms within a material. A type of convection, mostly applied to heat transfer in solid materials.

What is advection?



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Diffusion equation

$$q = -\rho\kappa\frac{\partial h}{\partial x}$$

$$\frac{\partial h}{\partial t} = -\frac{1}{\rho}\frac{\partial q}{\partial x}$$

- Last week we were introduced to the **diffusion equation**
  - Flux (transport of mass or transfer of energy) <u>proportional to a gradient</u>
  - Conservation of mass: <u>Any change in flux results in a change in mass/energy</u>

Diffusion equation

$$q = -\rho\kappa \frac{\partial h}{\partial x}$$

$$\frac{\partial h}{\partial t} = -\frac{1}{\rho}\frac{\partial q}{\partial x}$$

Diffusion

$$\frac{\partial h}{\partial t} = -\kappa \frac{\partial^2 h}{\partial x^2}$$

- Substitute the upper equation on the left into the lower to get the classic **diffusion equation**

  - q = sediment flux per unit length

    ρ = bulk sediment density

    κ = sediment diffusivity

    h = elevation

    x = distance from divide

    t = time

Advection and diffusion equations

Diffusion

$$\frac{\partial h}{\partial t} = -\kappa \frac{\partial^2 h}{\partial x^2}$$

Advection

$$\frac{\partial h}{\partial t} = c \frac{\partial h}{\partial x}$$

- This week we meet the **advection equation**

Advection and diffusion equations

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- This week we meet the **advection equation**
- Two key differences:
  - Change in mass/energy with time proportional to gradient, rather than curvature (or *change* in gradient)
  - **Advection coefficient** c has units of $[L/T]$, rather than $[L^2/T]$

Advection and diffusion equations

River channel profiles

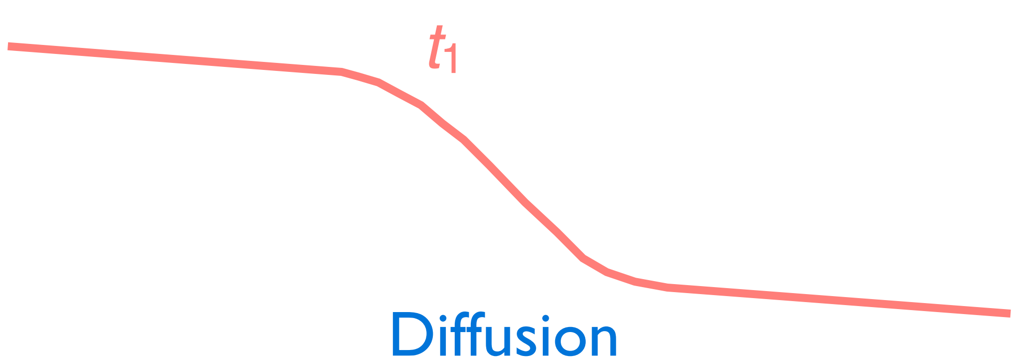

Fig. 1.7, Pelletier, 2008

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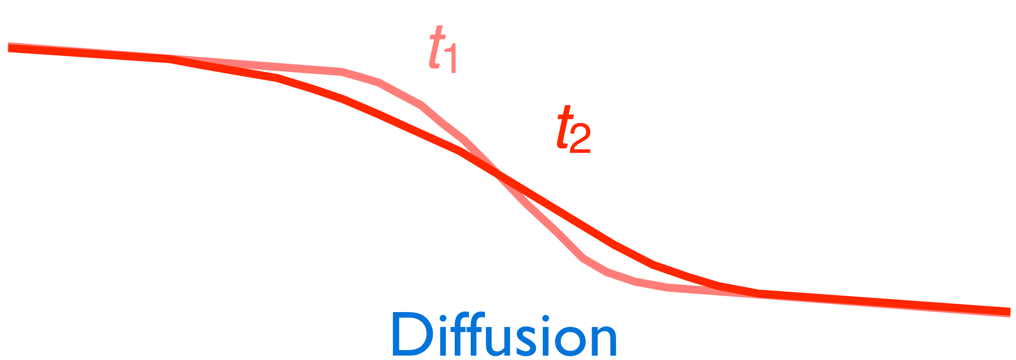

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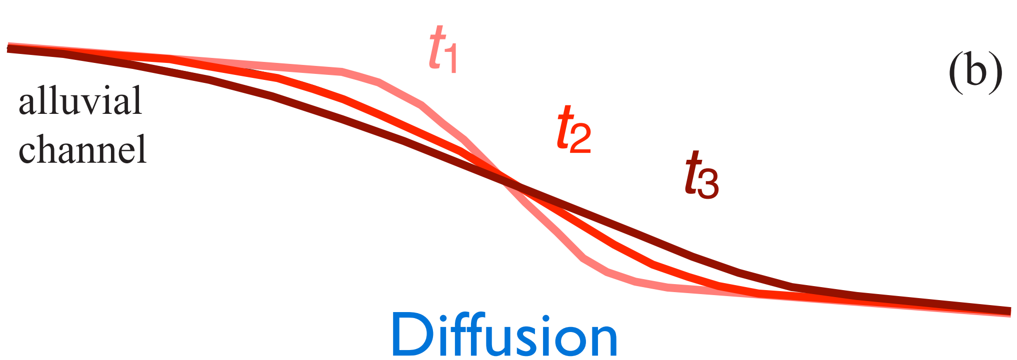

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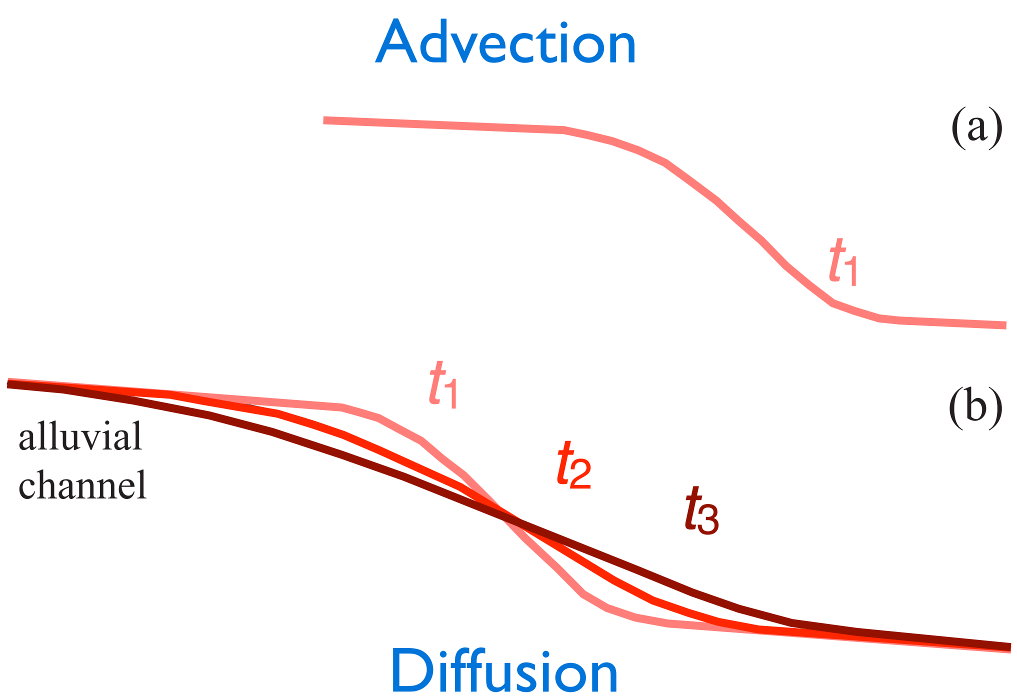

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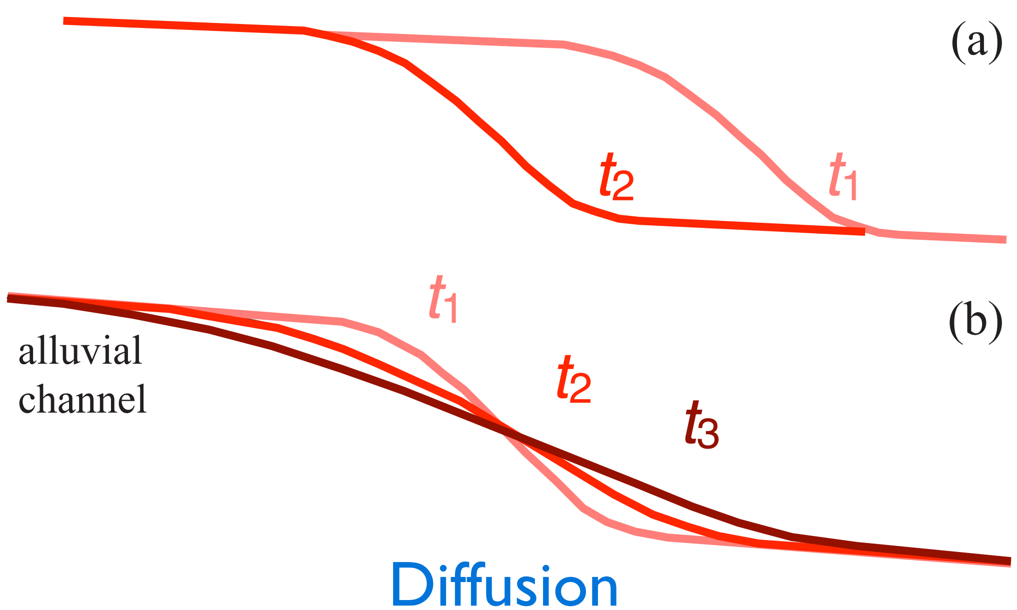

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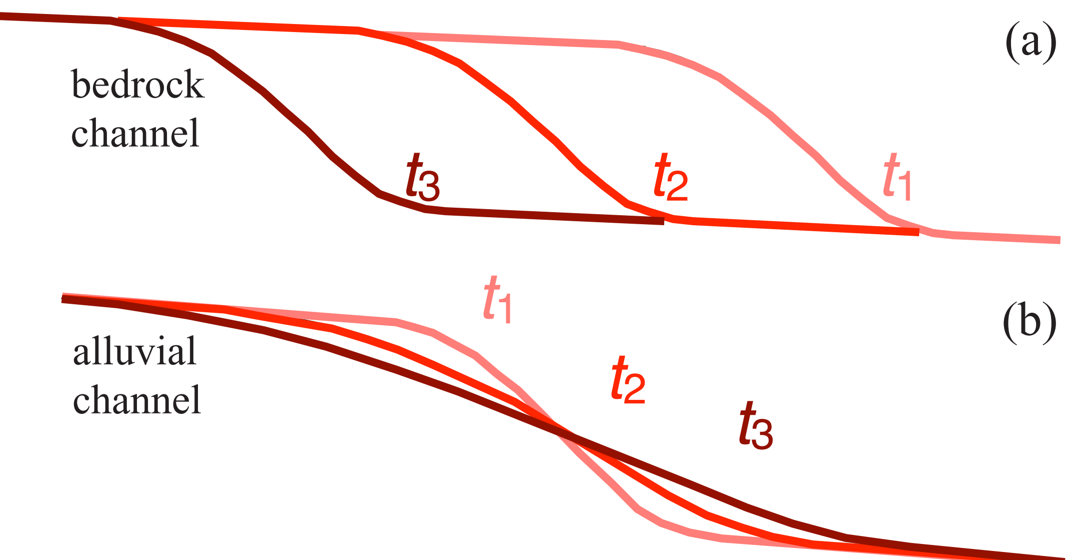

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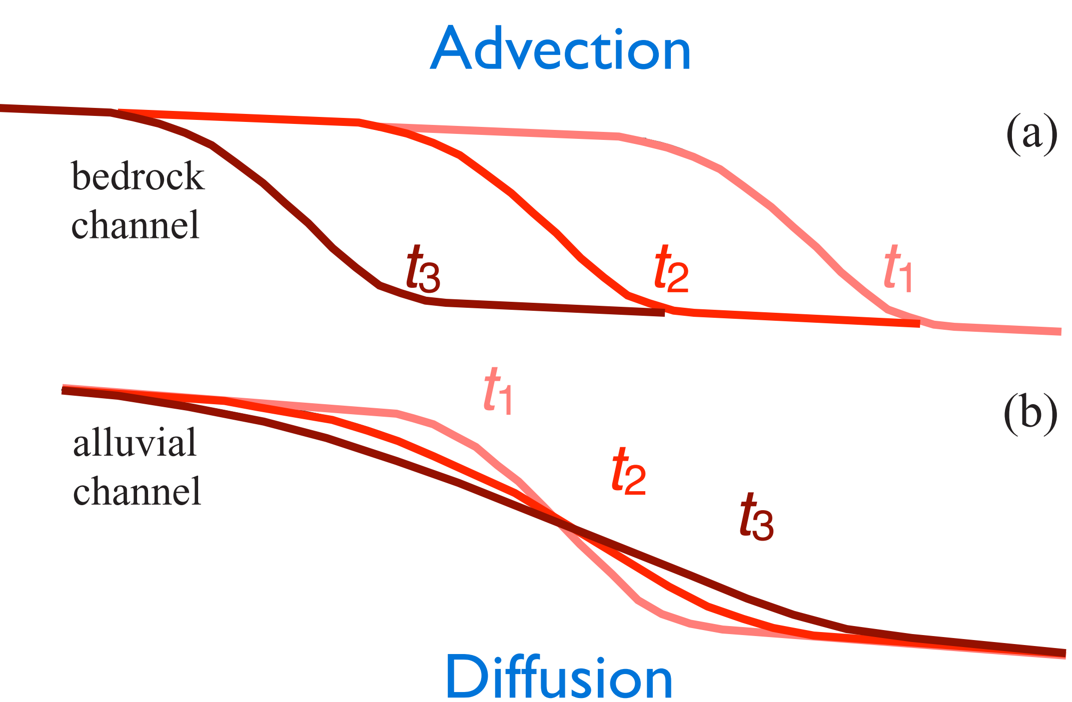

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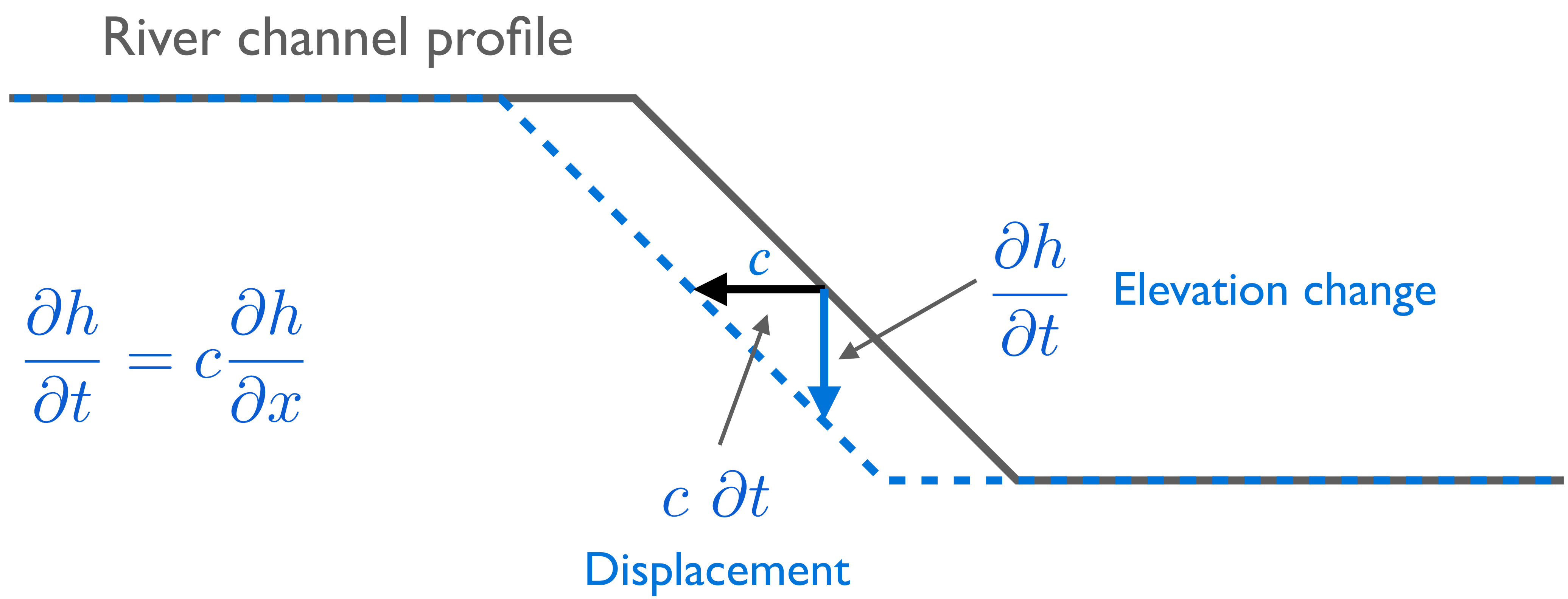
- **Diffusion**: Rate of erosion depends on change in hillslope gradient (curvature)
- **Advection**: Rate of erosion is directly proportional to hillslope gradient
  - Also, no conservation of mass (deposition)

Advection at a constant rate c

$$\frac{\partial h}{\partial t} = c\frac{\partial h}{\partial x}$$



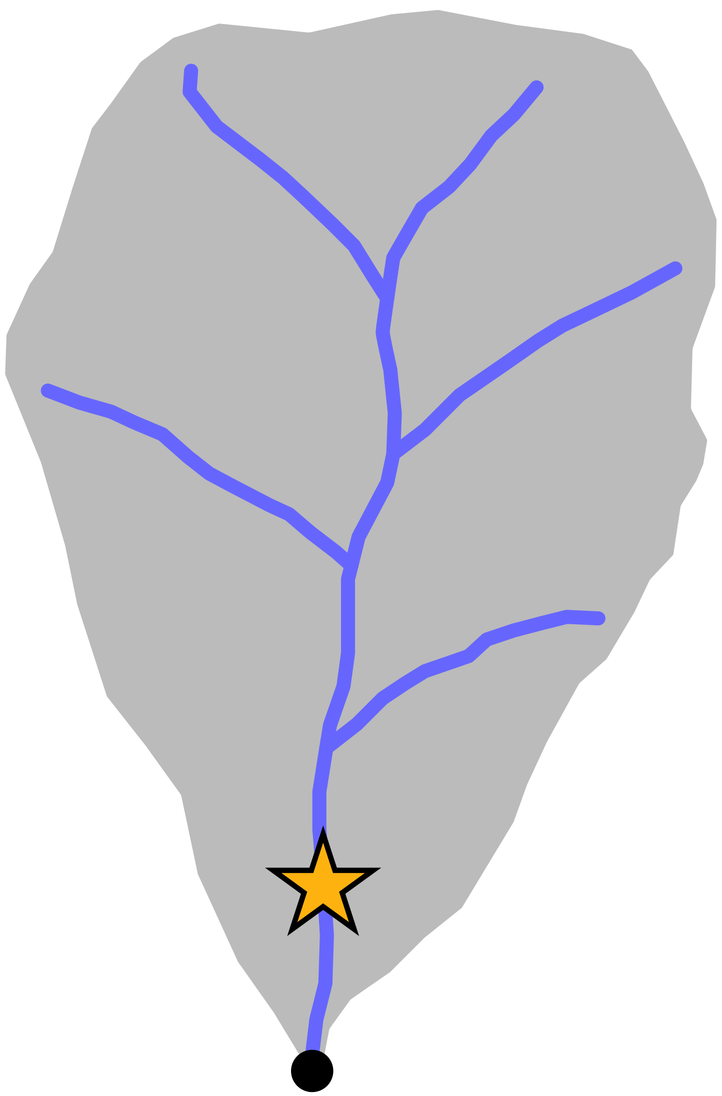
- Surface elevation changes in <u>direct proportion to surface slope</u>
- Result is lateral propagation of the topography or river channel profile
- Although this is interesting, it is <u>not that common in nature</u>

Advection of the Earth's surface: An example

- **Bedrock river erosion**
    - Purely an advection problem with a spatially variable advection coefficient

~~Bedrock~~ river erosion

Drainage basin



- Not much bedrock being eroded here…

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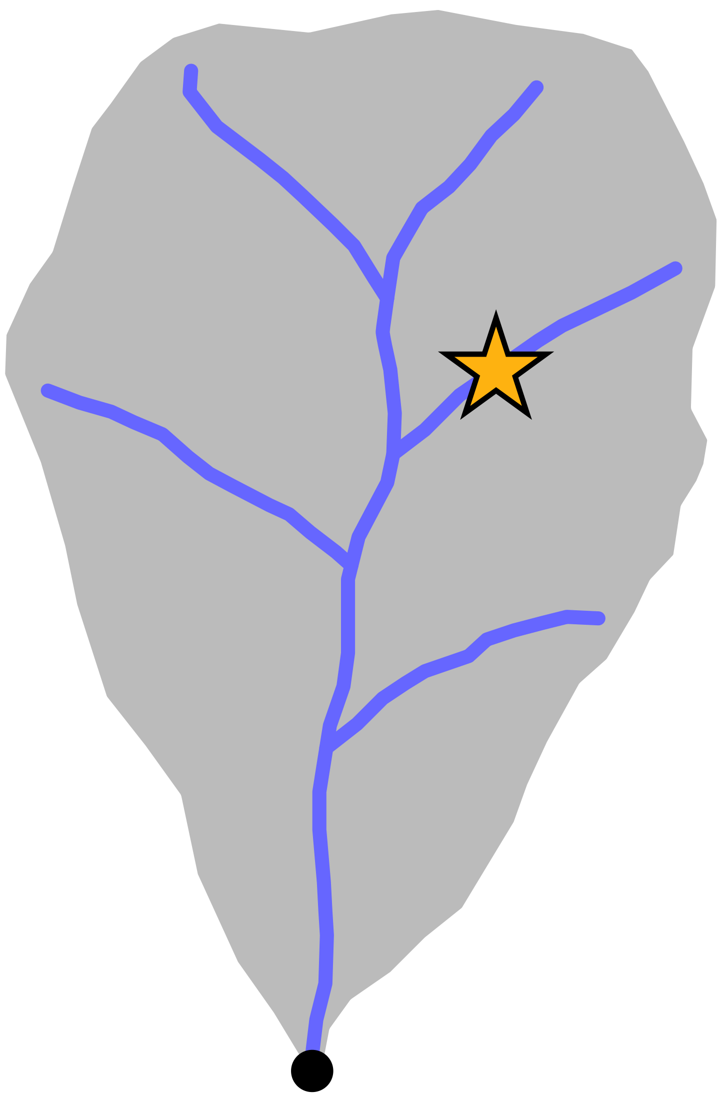
Bedrock river erosion

Drainage basin



Kali Gandaki river gorge, central Nepal
http://en.wikipedia.org/

- Rapid bedrock incision has formed a steep gorge in this case

River erosion as an advection process

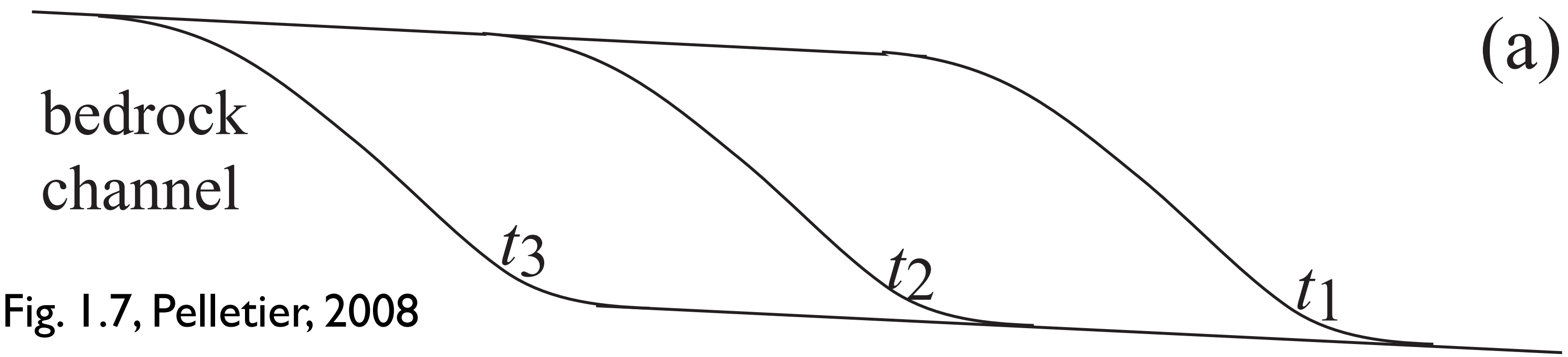
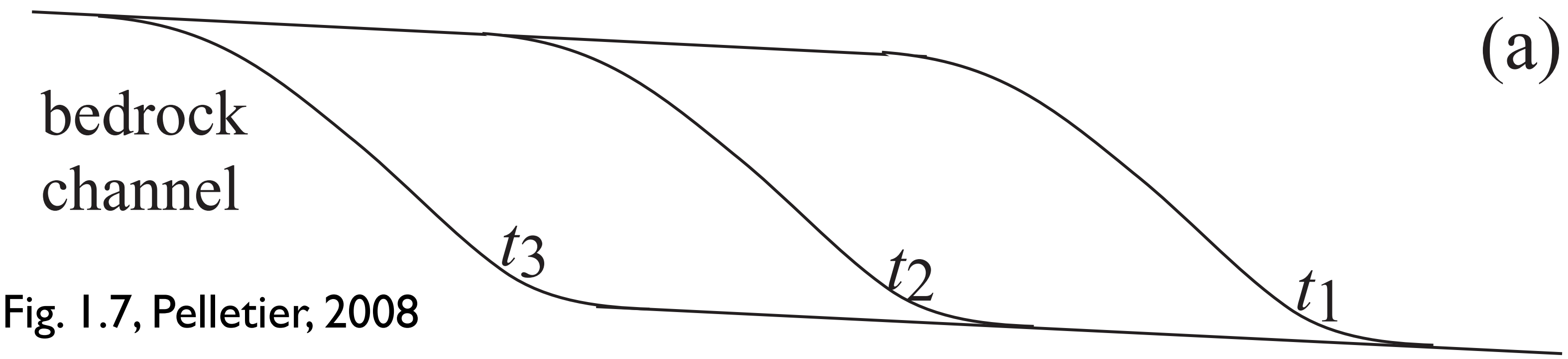


Fig. 1.7, Pelletier, 2008

- With a constant advection coefficient c, we predict lateral migration of the river profile at a constant rate (c)

River erosion as an advection process



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- With a constant advection coefficient c, we predict lateral migration of the river profile at a constant rate (c)
  - **Do you think this works in real (bedrock) rivers?**

River erosion as an advection process

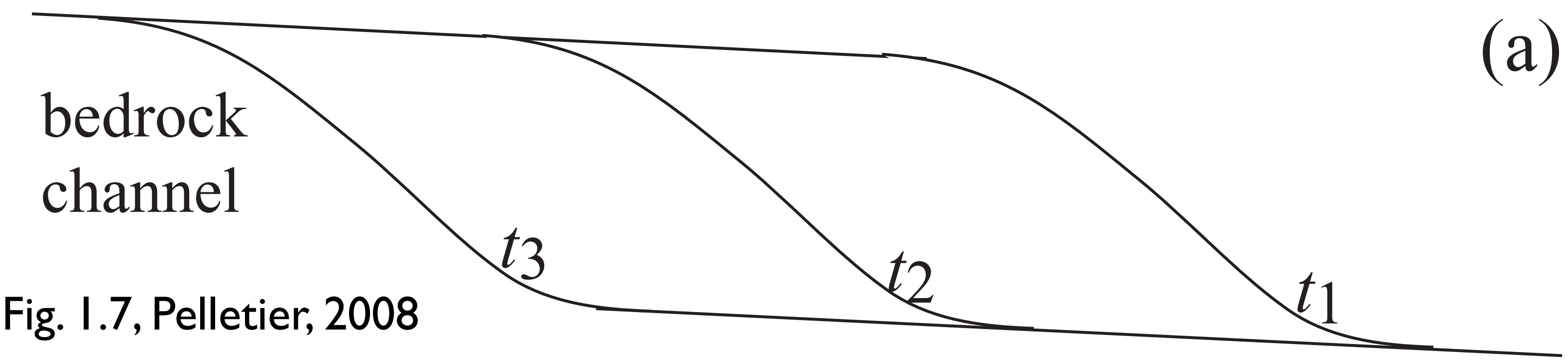

Fig. 1.7, Pelletier, 2008

- With a constant advection coefficient c, we predict lateral migration of the river profile at a constant rate (c)
  - **Do you think this works in real (bedrock) rivers?**
  - **What might affect the rate of lateral migration?**

What affects the efficiency of river erosion?

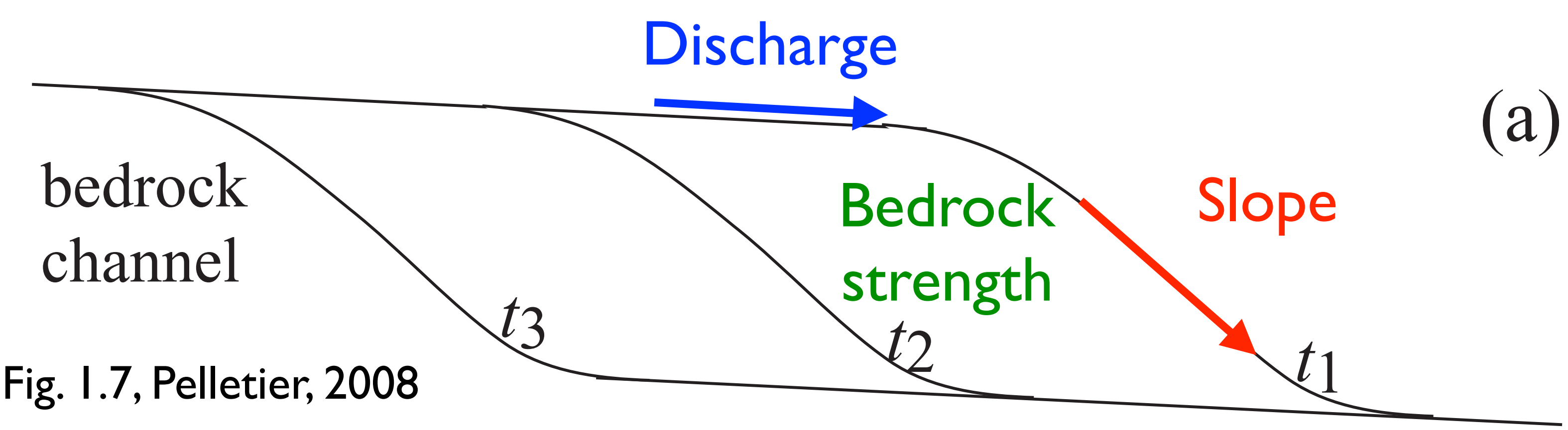

Fig. 1.7, Pelletier, 2008

- The amount of water flowing in the river (discharge) and sediment
- The slope of the river channel
- The strength of the underlying bedrock

What affects the efficiency of river erosion?

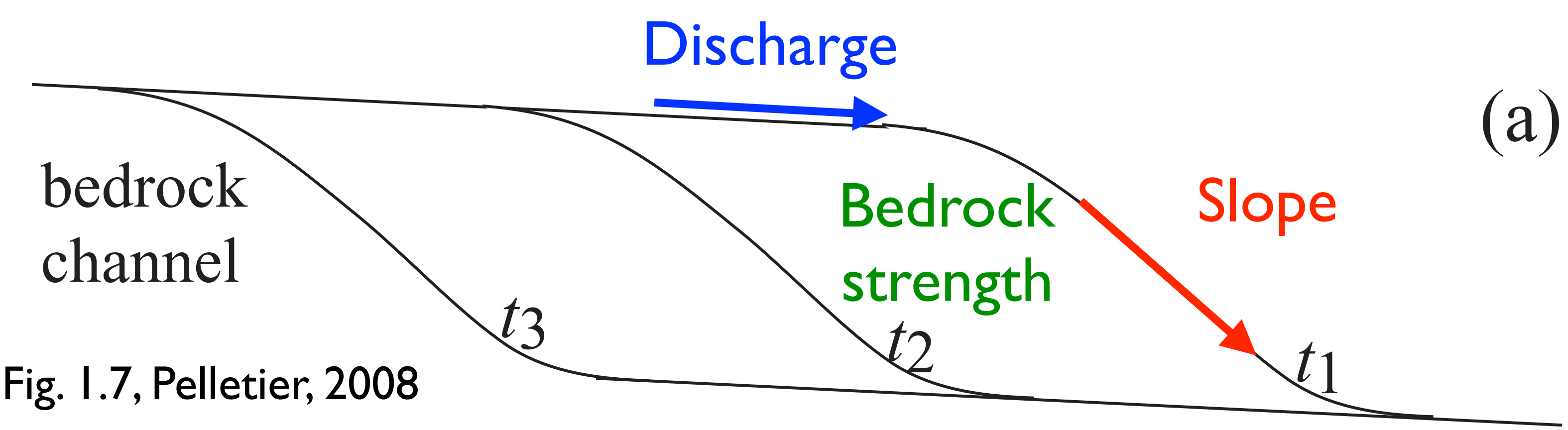

Fig. 1.7, Pelletier, 2008

- The amount of water flowing in the river (discharge) and sediment
- The slope of the river channel
- The strength of the underlying bedrock

- **Are these constant?**

Stream-power model of river incision

$$\cancel{\frac{\partial h}{\partial t} = c\frac{\partial h}{\partial x}}$$

- Rather than being constant, the rate of lateral advection in river systems is <u>spatially variable</u>

$$\frac{\partial h}{\partial t} = \frac{k_f}{w}Q\frac{\partial h}{\partial x}$$

where k_f is a material property of the bedrock (erodibility), w is the channel width, and Q is discharge

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- This is known as the **stream-power erosion model**

Stream-power model of river incision

- If we assume precipitation is uniform in the drainage basin, discharge Q will scale with drainage basin area, so we can modify our equation to read

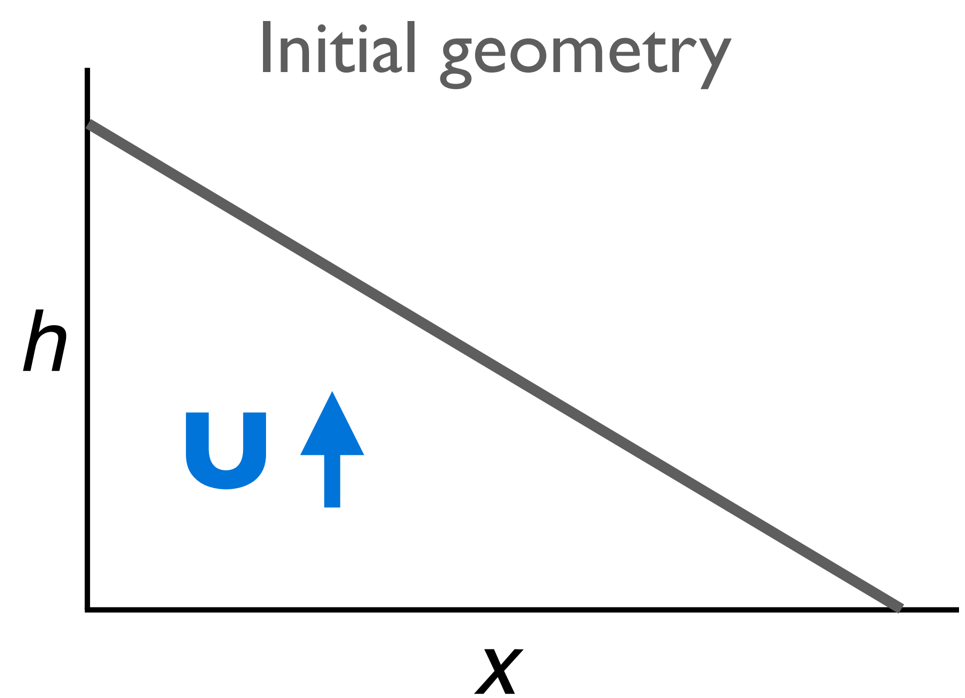
$$\frac{\partial h}{\partial t} = \frac{k_f}{w} Q \frac{\partial h}{\partial x} \longrightarrow \frac{\partial h}{\partial t} = K A^m S^n$$

where K is an erosional efficiency factor (accounts for lithology, climate, channel geometry, sediment supply, etc. (!)), A is upstream drainage area, S is channel slope, and m and n are area and slope exponents

- If we assume the drainage basin area increases with distance from the drainage divide x, we can replace the area with an estimate $A = x^{5/3}$

Test your might

$$\frac{\partial h}{\partial t} = U - KA^m S^n$$



- Based on our **stream-power erosion** equation, what general form would a channel profile take?
  - If we assume we have reached a steady state ($\partial h/\partial t = 0$) and $n = 1$, erosion must balance uplift U everywhere
  - If we further assume precipitation is constant, bedrock erodibility is constant and $A = x^{5/3}$, how would the channel steepness vary as you move downstream from the divide?
  - Think about how S would change as x increases

Evolution of a channel profile

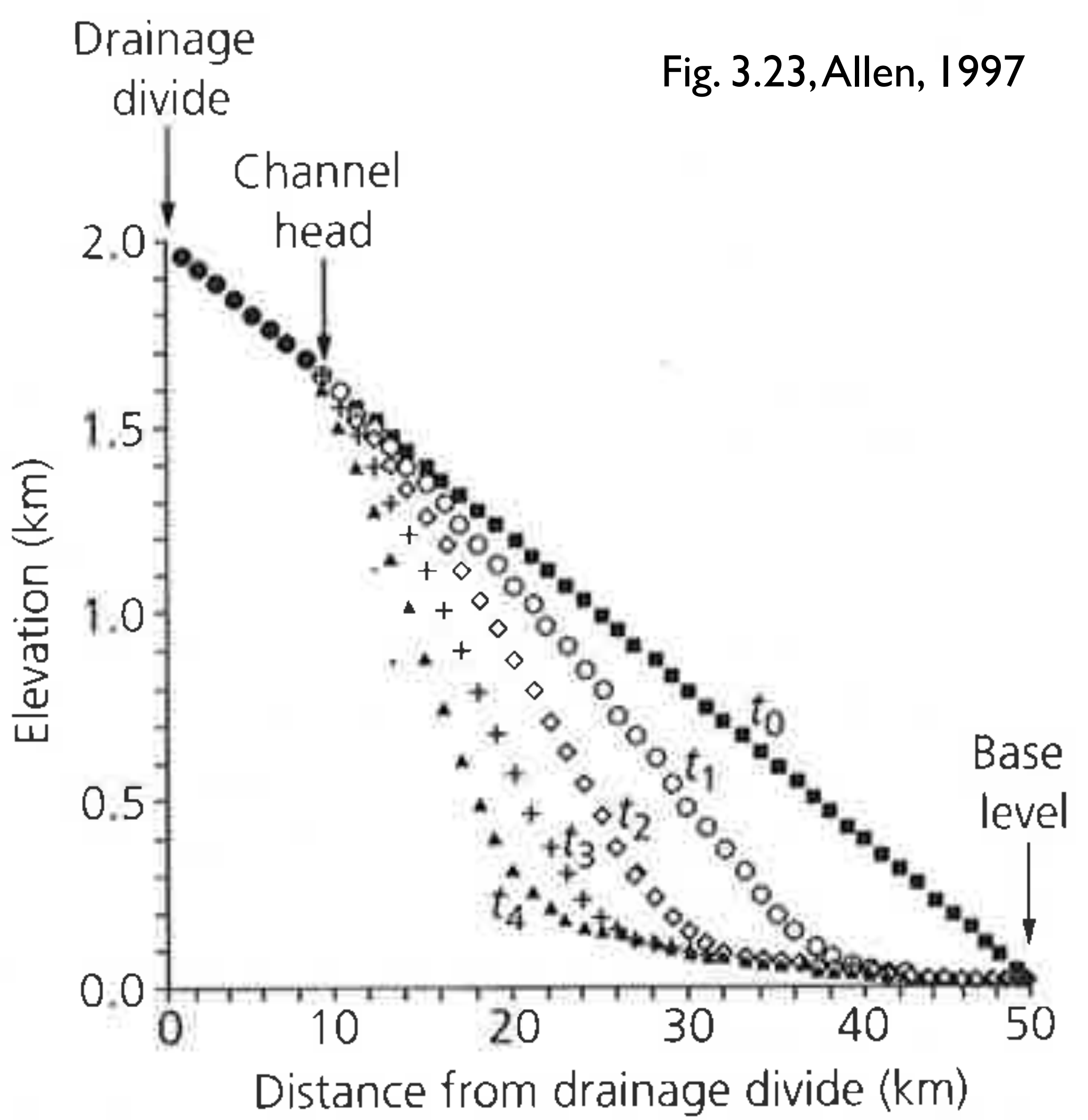


Fig. 3.23, Allen, 1997

- A few stream-power erosion observations:
  - Stream power increases downstream as the discharge grows
  - Steeper slopes occur upstream where the discharge is low
  - Incision migrates upstream until a balance is attained between erosion and uplift

Recap

- What is the main difference between the advection and diffusion equations?
- What is special about the stream power erosion model compared to the general advection equation?

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References

Allen, P.A. (1997). *Earth Surface Processes* (First edition.). Wiley-Blackwell.

Pelletier, J. D. (2008). *Quantitative modeling of earth surface processes* (Vol. 304). Cambridge University Press.