



Class overview today - November 26, 2018

- Lecture: **Rocks and ice as viscous materials**
 - Linear viscous flow
 - End-member types of linear viscous flows
 - Nonlinear viscosity

- Exercise 5: **Viscous flow of ice**



Introduction to Quantitative Geology

Rock and ice as viscous materials

Lecturer: David Whipp
david.whipp@helsinki.fi

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Goals of this lecture

- Introduce the basic relationship for **viscous flow** of rock and ice
- Explore two different end-member types of **viscous flow in a channel**
- Discuss the effects of **temperature on viscosity** and **nonlinear viscosity**



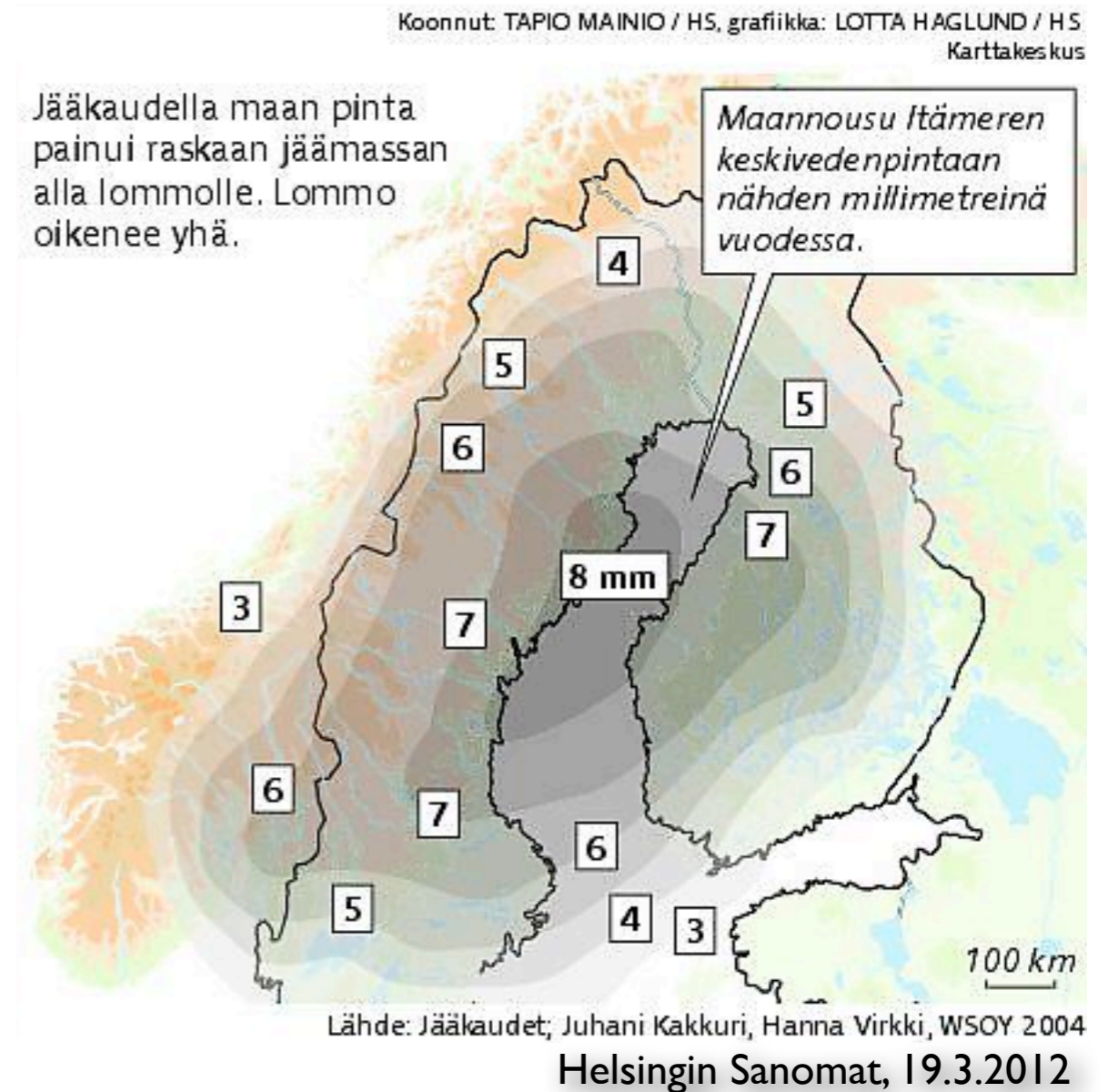
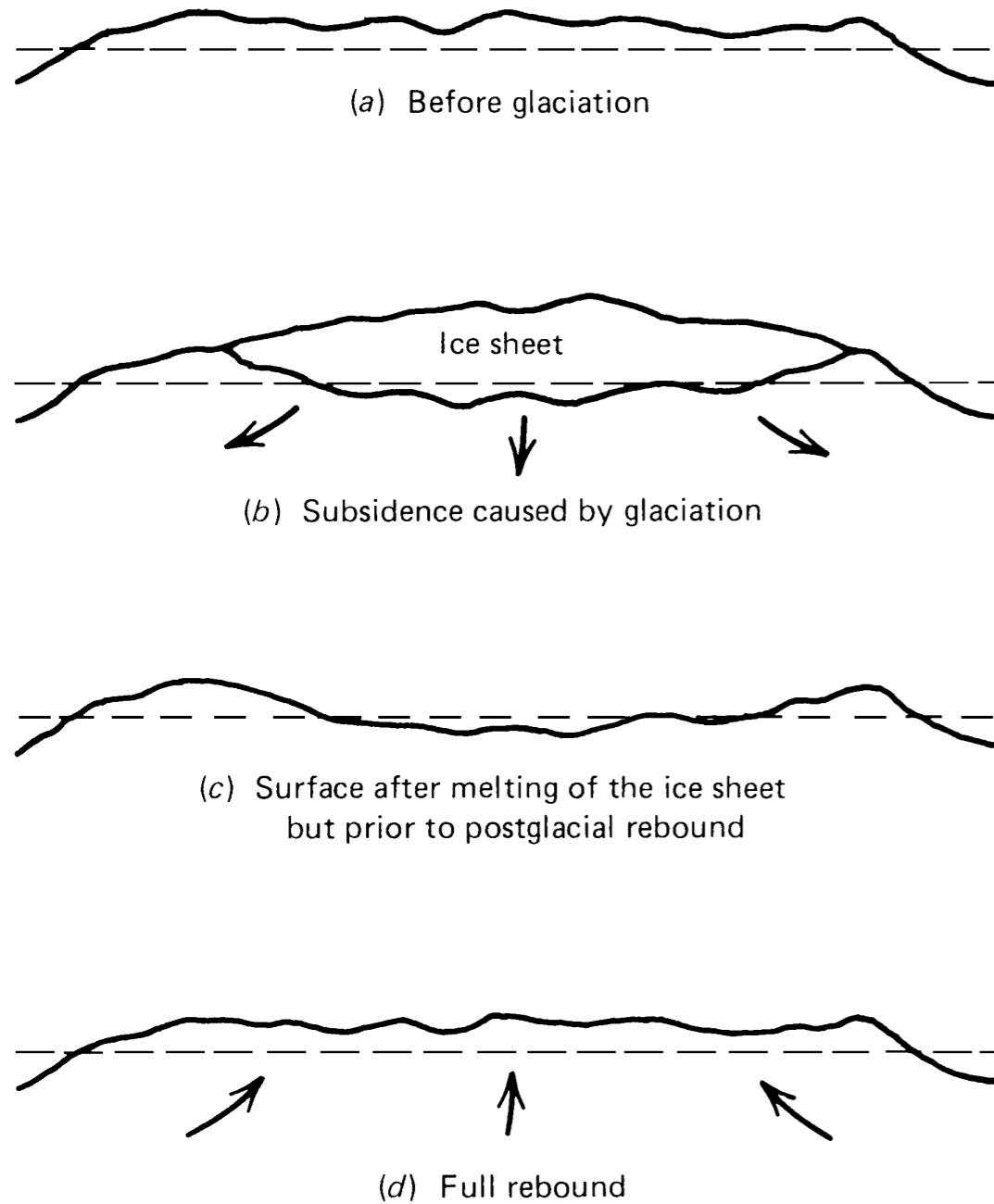
Examples of viscous flow: Alpine glaciers



- **Alpine glaciers flow downhill** under their own weight



Glacio isostatic adjustment



Surface uplift due to glacio isostatic adjustment is controlled by **flow of the underlying asthenosphere**



What is a fluid?

- **Fluid:** Any material that flows in response to an applied stress
- Deformation is continuous
- Stress is proportional to strain rate

$$\tau \propto \frac{du}{dz}$$

where τ is the **shear stress**, du/dz is the **velocity gradient** (equivalent to strain rate) and u is the **velocity in the x -direction**

Viscosity, defined

Low viscosity



High viscosity



- Constant of proportionality η is known as the **dynamic viscosity**, or often simply **viscosity**

$$\text{I-D: } \tau = \eta \frac{du}{dz}$$

- **Viscosity** has units of **Pa s** (Pascal seconds) or **kg m⁻¹ s⁻¹**
- You can think of viscosity as a resistance to flow
- Higher viscosity → more resistant to flow, and vice versa
- The terms **kinematic viscosity** and **bulk viscosity** (or compressibility) are not the same thing as the dynamic viscosity

<http://en.wikipedia.org>



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Approximate viscosities of common materials

Material	Viscosity [Pa s]
Air	10^{-5}
Water	10^{-3}
Honey	10^1
Basaltic lava	10^3
Ice	10^{10}
Rhyolite lava	10^{12}
Rock salt	10^{17}
Granite	10^{20}



A honey dipper works because of the viscosity of honey

- Viscosity of natural materials is hugely variable
- Range of almost 20 orders of magnitude for rocks and lava



Newtonian (linear) viscosity

$$\tau = \eta \frac{du}{dz}$$

- A **Newtonian material** has a linear relationship between shear stress and strain rate
- In other words, η is a constant value that does not depend on the stress state or flow velocity
- Air, water and thin motor oil are practically Newtonian fluids
- Rocks rarely deform as Newtonian fluids



Linear viscous flow in a channel

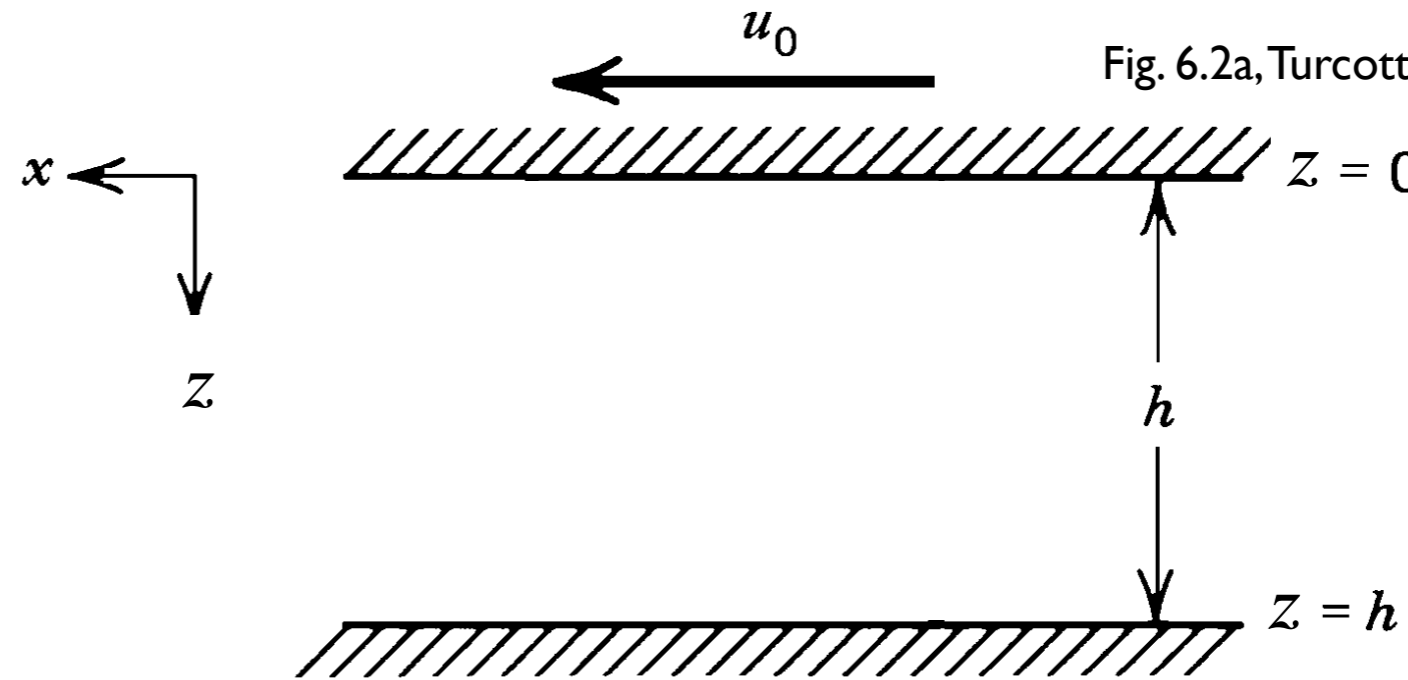


Fig. 6.2a, Turcotte and Schubert, 2014

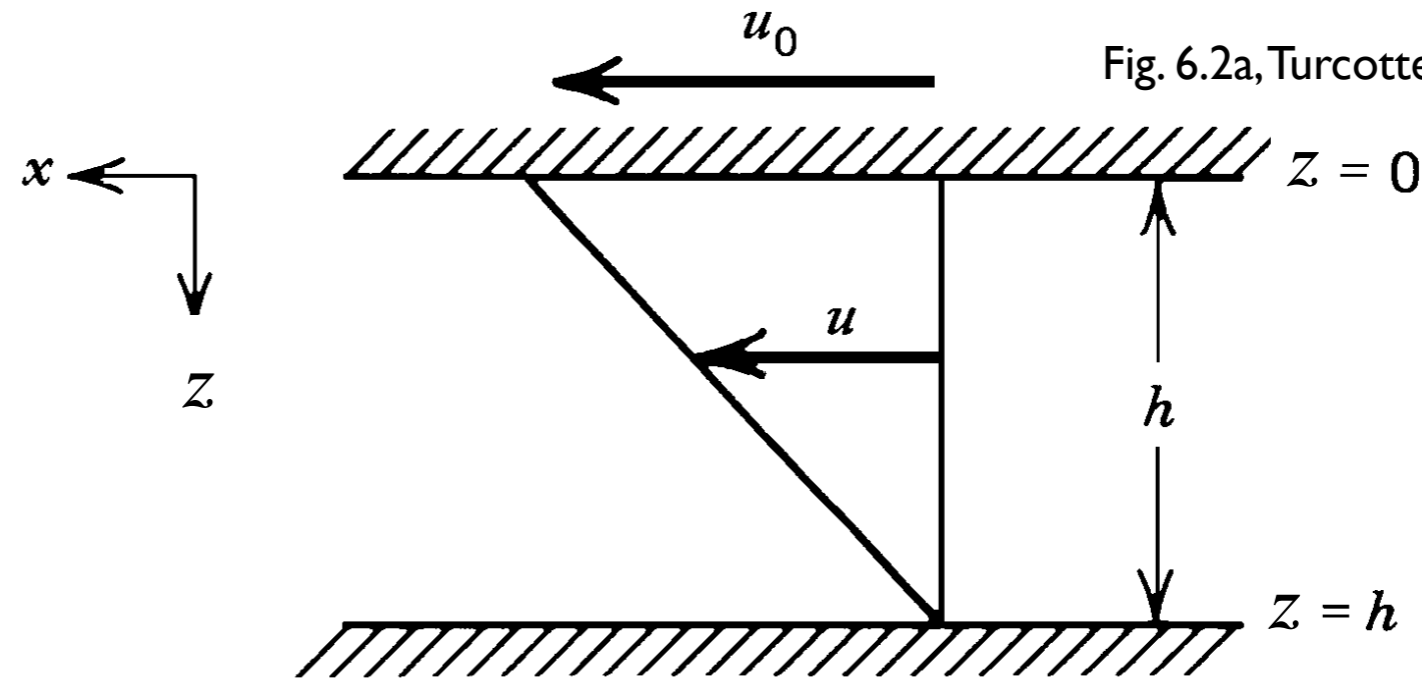
- The general solution for the 1-D velocity of a fluid across a channel with boundary conditions (1) $u = 0$ at $z = h$ and

(2) $u = u_0$ at $z = 0$ is

$$u = \frac{1}{2\eta} \frac{dp}{dx} (z^2 - hz) - \frac{u_0 z}{h} + u_0$$



Styles of linear viscous flow: Couette flow

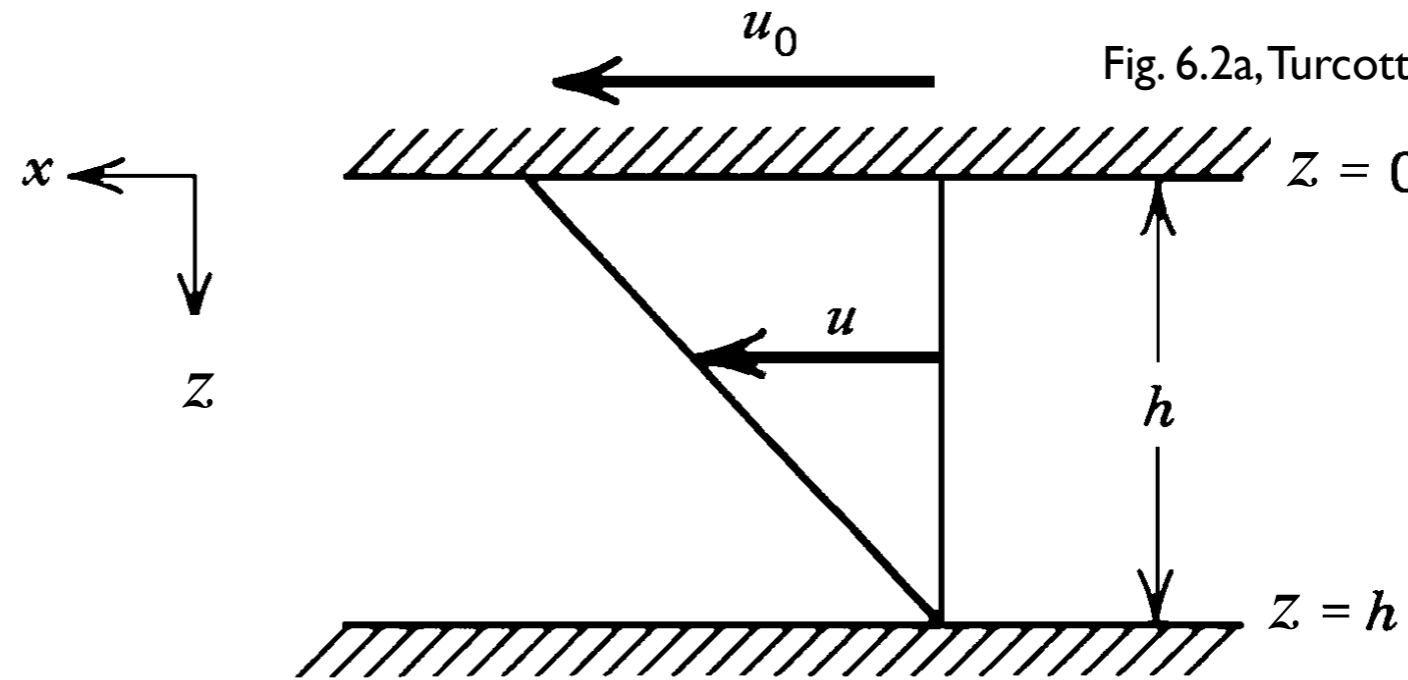


(a) Couette flow $\frac{dp}{dx} = 0, u_0 \neq 0$

- **Couette flow** occurs when there is (1) a difference in velocity between the channel boundaries and (2) effectively no pressure gradient



Couette flow solution



(a) Couette flow $\frac{dp}{dx} = 0, u_0 \neq 0$

- If we assume $dp/dx = 0$,

$$u = \frac{1}{2\eta} \frac{dp}{dx} (z^2 - hz) - \frac{u_0 z}{h} + u_0$$

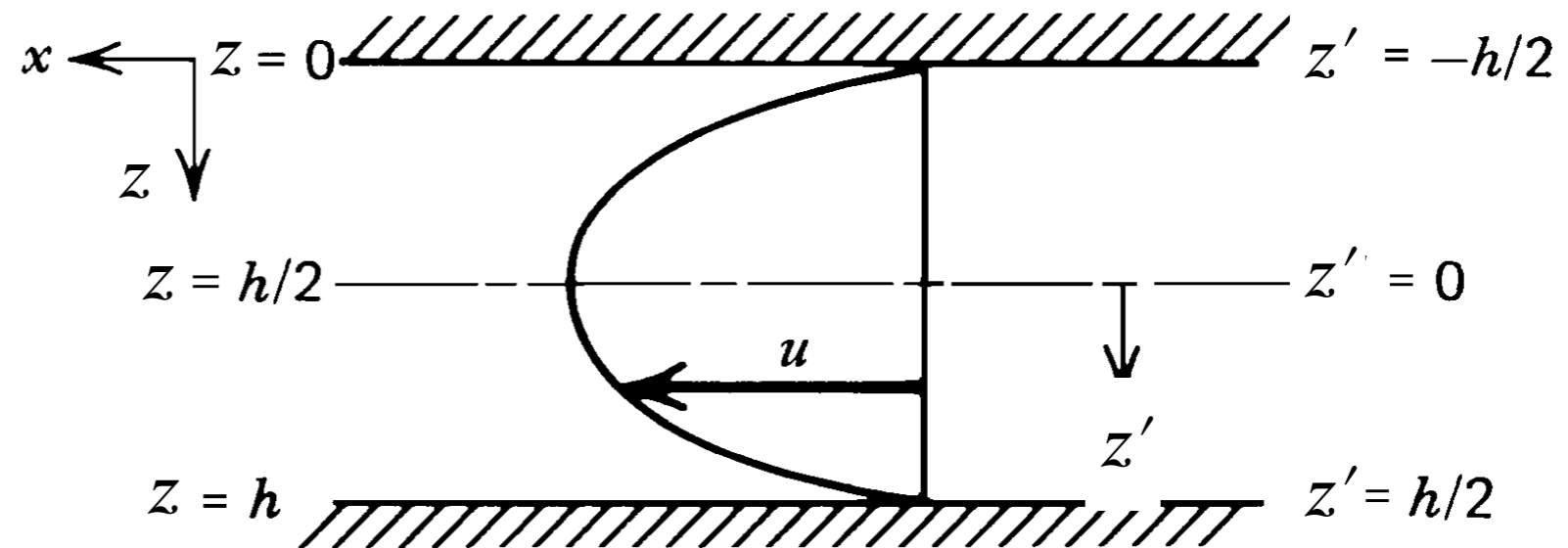
reduces to

$$u = u_0 \left(1 - \frac{z}{h} \right)$$



Poiseuille flow

Fig. 6.2b, Turcotte and Schubert, 2002



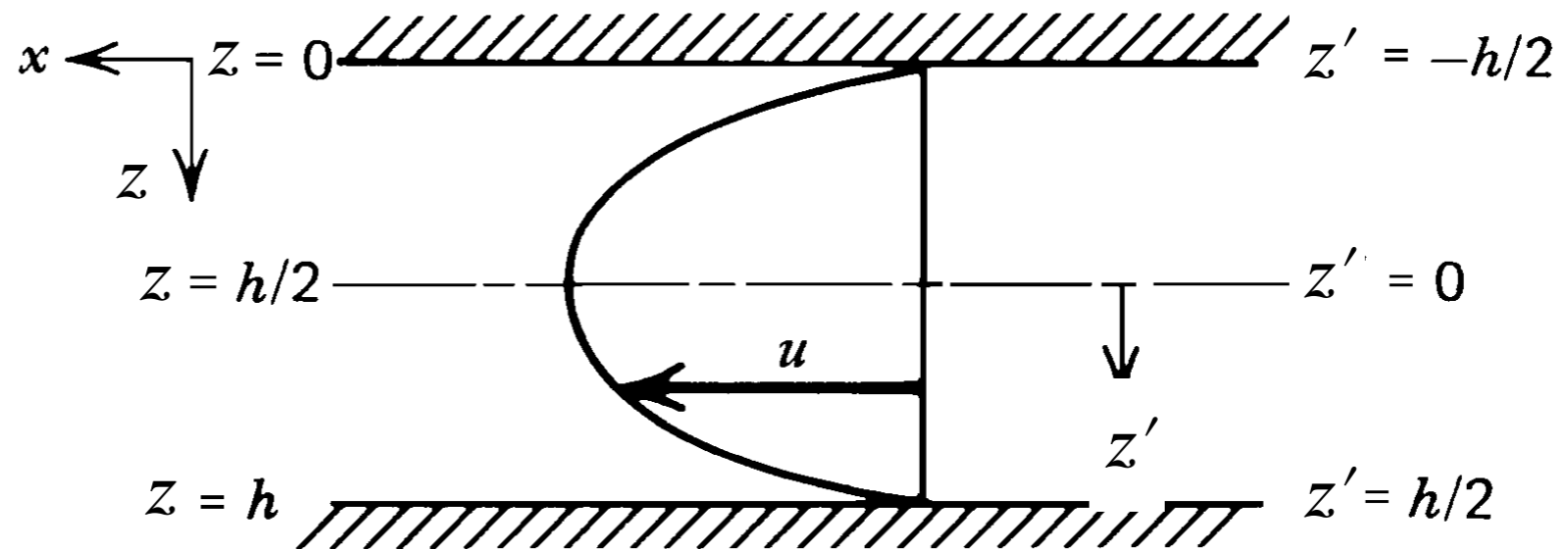
$$(b) \frac{dp}{dx} \neq 0, u_0 = 0$$

- **Poiseuille flow** occurs when (1) there is no velocity difference between the walls of the channel and (2) a pressure gradient is applied



Poiseuille flow solution

Fig. 6.2b, Turcotte and Schubert, 2002



$$(b) \frac{dp}{dx} \neq 0, u_0 = 0$$

- Using the same equation as we have previously, we can start with the general solution

$$u = \frac{1}{2\eta} \frac{dp}{dx} (z^2 - hz) - \frac{u_0 z}{h} + u_0$$

- If we set $u_0 = 0$, the velocity solution becomes

$$u = \frac{1}{2\eta} \frac{dp}{dx} (z^2 - hz)$$



Salt tectonics

<http://commons.wikimedia.org>



Head of salt diapir

- One example of a geological system that can exhibit both **Couette** and **Poiseuille** flow behavior is the flow of rock salt beneath sedimentary overburden



Temperature dependence

- In general, rock viscosity depends strongly temperature

$$\eta = A_0 e^{Q/RT_K}$$

where A_0 and Q are material properties known as the **pre-exponent constant** and **activation energy**, R is the **universal gas constant** and T_K is **temperature in Kelvins**



Temperature-dependent viscosity

- The viscous strength of quartz, for example, rapidly decreases with increasing temperature
- Note that the viscous strength is simply the viscosity η multiplied by a nominal strain rate

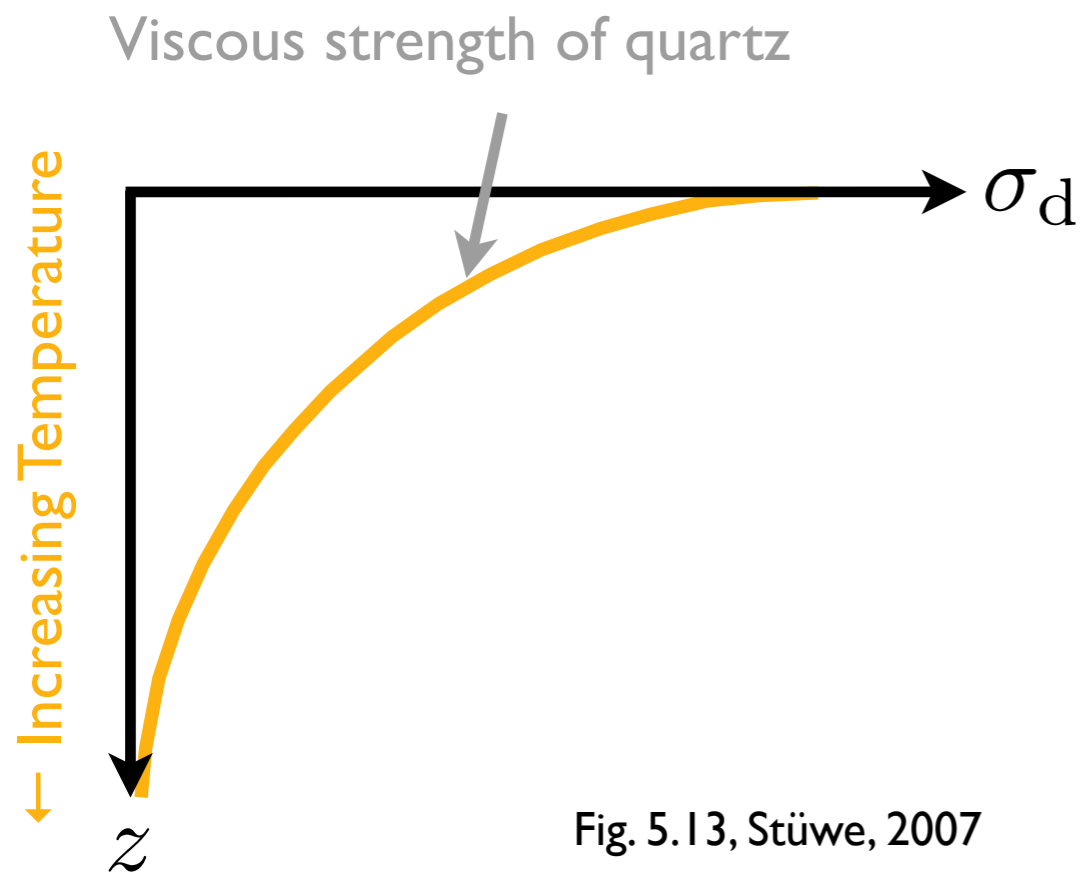


Fig. 5.13, Stüwe, 2007



Temperature-dependent viscosity

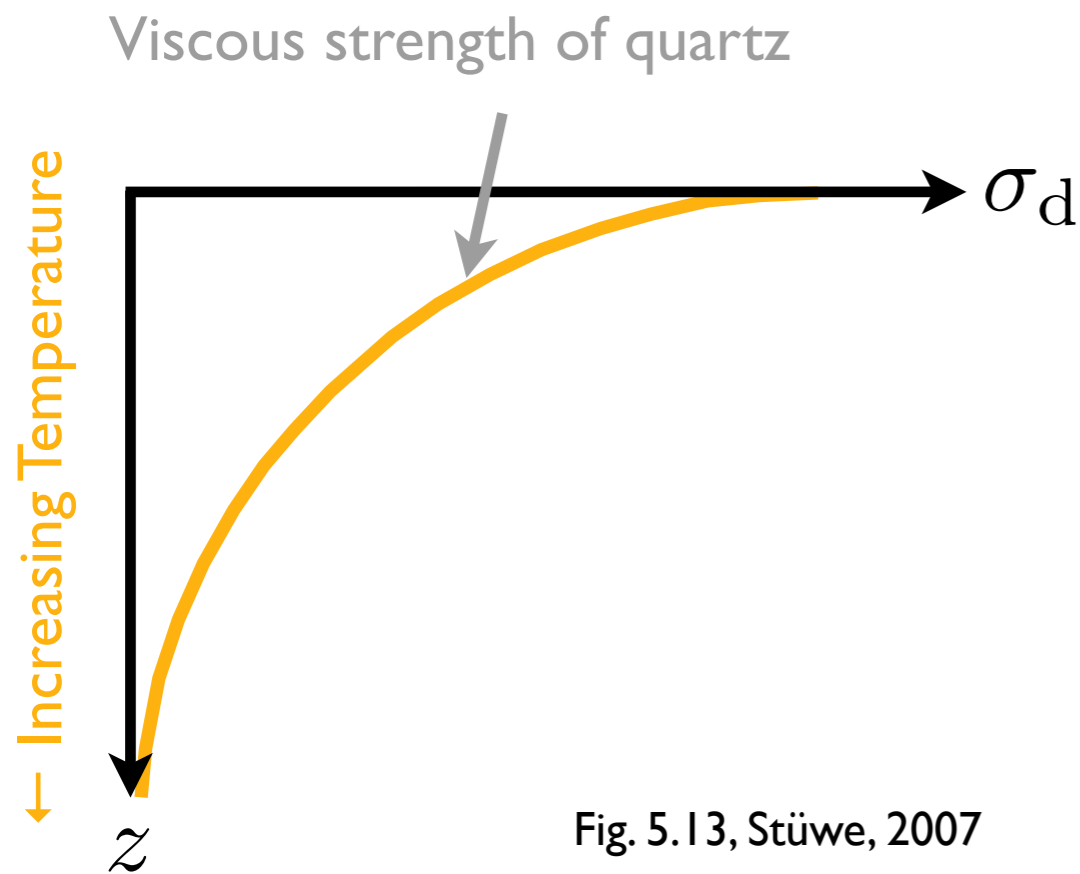


Fig. 5.13, Stüwe, 2007

- The viscous strength of quartz, for example, rapidly decreases with increasing temperature
- Note that the viscous strength is simply the viscosity η multiplied by a nominal strain rate
- **How might temperature-dependent viscosity be important in the Earth?**



Nonlinear viscosity

- In general, rocks will deform about 8 times as quickly when the applied force is doubled
- Relationship between shear stress and strain rate is thus **NOT linear**
- Mathematically, we can say

$$\tau^n = A_{\text{eff}} \frac{du}{dz}$$

where n is the **power law exponent** and A_{eff} is a **material constant**

- The power law exponent for many rocks is 2-4
- A_{eff} is similar to η , but has units of **$\text{Pa}^n \text{ s}$**



Flow of glaciers

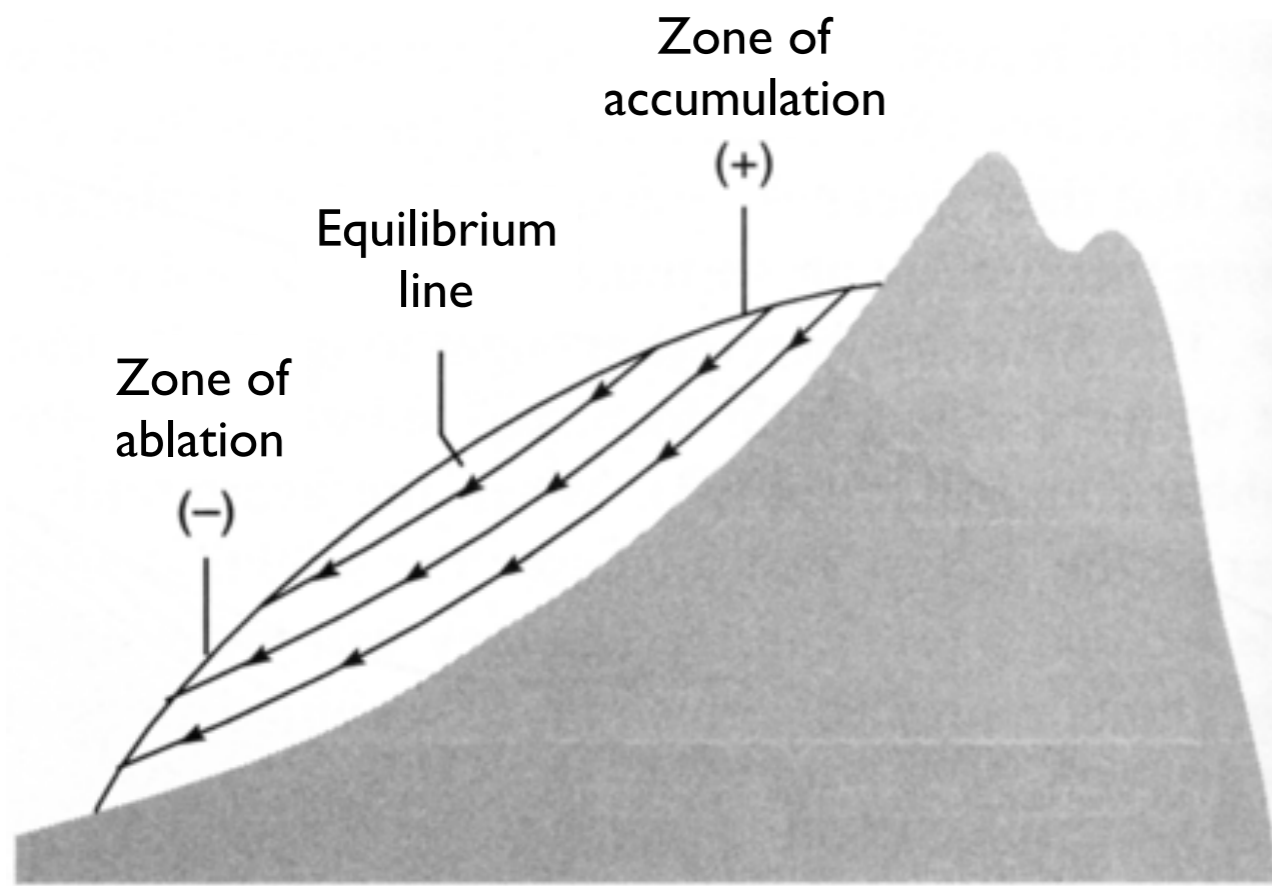


Fig. 9.14, Ritter et al., 2002

- Gravity drives the flow of alpine glaciers from higher elevation zones of **accumulation** to lower elevation zones of **ablation**
- Depending on the temperature of the region and the ice itself, the glacier may either be frozen to the bedrock (**cold-based**) or sliding along the bedrock (**warm-based**)



How do glaciers move?



Briksdal Glacier, Norway

- **Basal sliding**
 - Bottom of the glacier sliding along the substrate
 - Can occur as a result of slip atop a thin water layer, melting/re-freezing or slip atop water-saturated sediment
- **Internal deformation**
 - Ice flow is nonlinear viscous and sensitive to temperature
 - Deformation is concentrated near the bed



Flow of glaciers

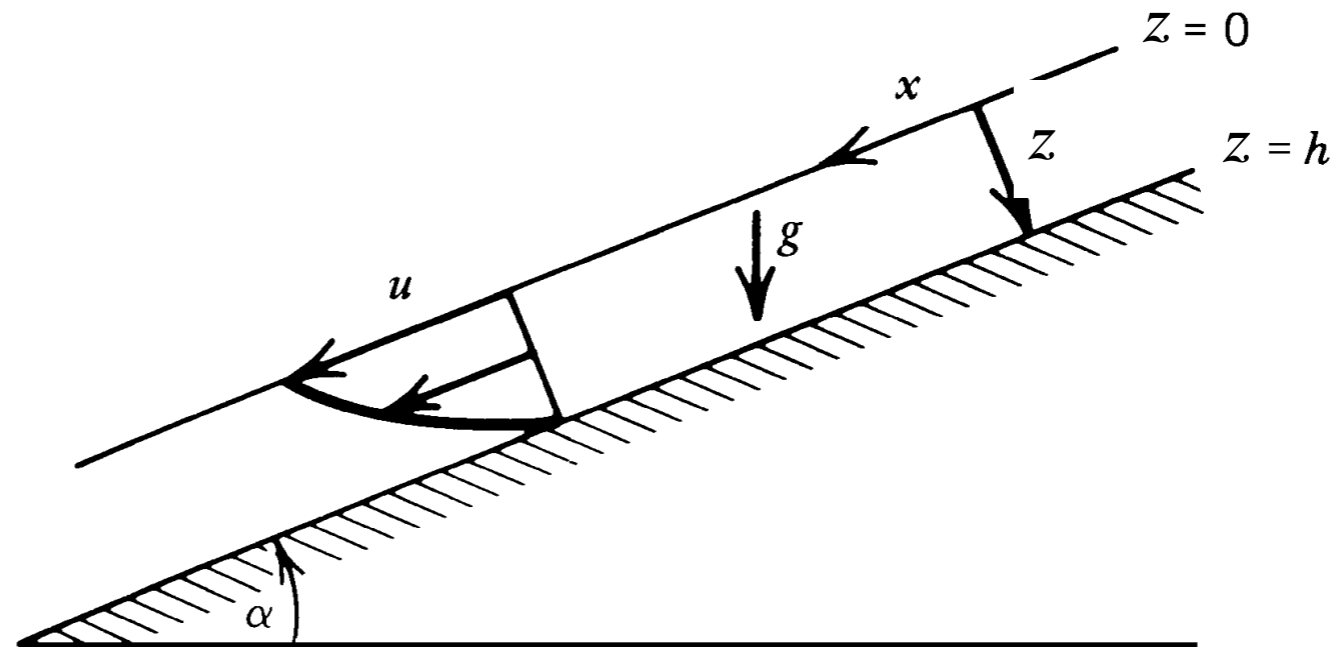


Fig. 6.3, Turcotte and Schubert, 2014

- In the exercise this week, we will look more closely at glacial flow
- Velocity across a glacial valley
- Down an incline



Recap

- **Viscous flow** is a common deformation behavior for rock and ice, where the deformation rate is proportional to the applied shear stress
- **Couette** and **Poiseuille** flows refer to end-member behaviors of linear viscous channel flows, and depend on the channel boundary velocities and pressure changes along the channel
- Most rocks do not exhibit a linear relationship between stress and strain rate (nonlinear viscosity), and their viscosity is strongly temperature-dependent



References

Ritter, D. F., Kochel, R. C., & Miller, J. R. (2002). *Process Geomorphology* (4 ed.). McGraw-Hill Higher Education.

Stüwe, K. (2007). *Geodynamics of the Lithosphere: An Introduction* (2nd ed.). Berlin: Springer.

Turcotte, D. L., & Schubert, G. (2014). *Geodynamics* (2nd ed.). Cambridge, UK: Cambridge University Press.